Cooper pair tunneling in junctions of singlet quantum Hall states and superconductors

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We propose tunnel junctions of a Hall bar and a superconducting lead, for observing Cooper-pair tunneling into singlet fractional quantum Hall edge states. These tunnel junctions provide a natural means of extracting precise information of the spin polarization and the filling factor of the state. The low energy regime of one of the set-ups is governed by a novel quantum entangled fixed point.

Junctions of fractional quantum Hall (FQH) states are unique tools to investigate fascinating and unexpected aspects of Quantum Mechanics. In this paper we discuss the physics of *singlet* FQH/superconductor (SC) junctions. Tunneling in these junctions involve Cooper pairs which either tunnel as a whole, or split into their constituent electrons which, if quantum coherence is not spoiled, must retain the spin-singlet nature of the Cooper pair and thus show quantum correlations in apparently separate physical systems.

The set-up we propose offers a direct means of probing the transition from spin-polarized to singlet QH (SQH) states observed for certain filling factors, such as $\nu = 2/5, \nu = 2/3$, driven either by application of hydrostatic pressure or by tilted magnetic fields [1, 2, 3, 4]. Furthermore, via enhanced tunneling conductance, it can provide precise information of effective filling factors by filtering out SQH states as a function of magnetic field. The beauty of this setup lies in the fact that the tuning of appropriate energy scales can turn the tunnel junction into an entanglement device wherein two disconnected QH droplets extract entangled electron pairs from the SC lead. Cooper pair tunneling and splitting are a focus of interest in the context of quantum information in solid state systems such as quantum dots and nanotubes [5]. In principle, the SC/SQH junctions discussed here have the advantage of bypassing fabrication issues present in these other set-ups, particularly with regards to the control of the contact at the junction. As we shall see, the low energy physics of the SQH/SC junctions is governed by a quantum entangled fixed point (QEFP) which links together the SC lead and two SQH edge states.

We are interested in tunneling of Cooper pairs from singlet-paired SC electrodes into SQH states sketched in Fig.1. So as to observe purely Cooper pair tunneling, and no Bogoliubov quasiparticle excitations, we assume all the important energy scales are smaller than the superconducting gap energy. Notice that the physics here is markedly different from that of conventional superconductor-insualtor-normal (S-I-N) junctions, since although all of the QH bulk excitations are gapped, the QH droplet supports gapless chiral edge excitations that provide a non-dissipative channel for conduction. Thus far, theoretical treatments of such a set-up have been relatively few [6, 7] as it requires a superconductor in high magnetic fields. However, experiments using NbN films
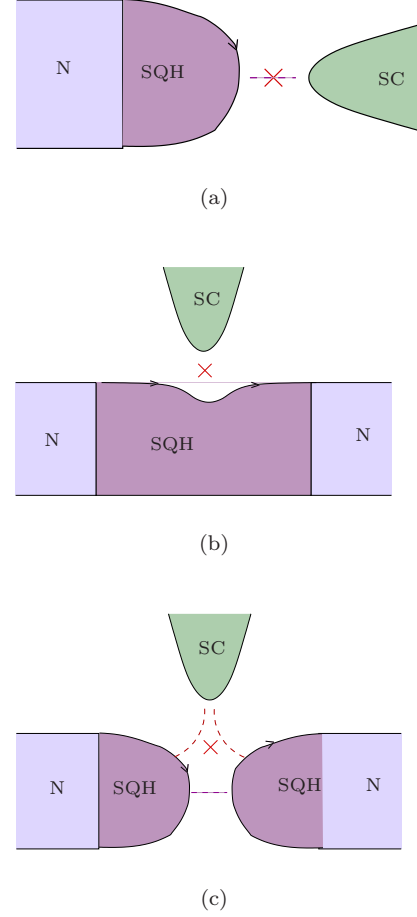

FIG. 1: Cooper pair tunneling from a SC to SQH state. (a) Set-up I, with one SQH and one SC lead; Set-up II, a SQH bar coupled to a SC lead in the (b) UV and (c) IR regimes. Notice the splitting of the SQH bar in (c).

as SC leads in contact with QH systems [8, 9] suggest a more practical alternative.

Fig.1(a) shows the simplest instance (Set-up I) of singlet Cooper pairs tunneling from a superconducting lead through a point contact (PC) into the singlet FQH state which in turn is in ohmic contact with a normal lead. Here we assume that the geometry of the FQH fluid is such that it is not significantly rearranged by

the proximity of the superconductor. Unlike tunneling of Cooper pairs into fully polarized Laughlin states in Ref.[6], the tunneling conductance into spin singlet FQH states would not be exponentially suppressed since in this case pair-breaking spin flip processes are not required. In set-up II, we consider a QH bar geometry which makes ohmic contact with two normal leads, and a tunnel contact with a SC wedge which acts as a third lead.

Figures 1(b) and (c) show the Set-up II in two different regimes of energy scale which depend on the experimental parameters (for example, temperature and bias voltage). In the high-voltage ("ultra-violet" (UV)) limit (Fig.1(b)), the allowed pair tunneling process is effectively the same as in Set-up I. However, in the low-voltage ("infra-red" (IR)) regime (Fig.1(c)) the presence of the SC lead induces backscattering processes among the edges of the QH bar which leads to a splitting of the FQH fluid into two droplets by quasiparticle tunneling[10]. We find that in this regime the most favored pair tunneling process is the one in which pairs from the SC lead split into the two droplets as entangled electrons. Hence, the low energy behavior of the junction is governed by a new type of fixed point, a *quantum entangled fixed point* (QEFP), characterized by a boundary condition which mixes the phases of the spin channels of the split singlet FQH fluids and the SC into a singlet state.

In what follows, we present the formalism for describing the FQH edge states, the SC degrees of freedom and tunneling between the two systems. In the SC lead the phase of the superconductor can be pinned or endowed with its own dynamics, and we account for both possibilities. We calculate the tunneling conductances for the processes shown in Fig.1 and discuss their behavior.

The starting point of all our analyses is a model Lagrangian for the FQH-SC junction that describes the dynamics of the FQH edge and the SC lead, and the tunneling between them through a single PC, given by

$$\mathcal{L} = \mathcal{L}_0^{FQH} + \mathcal{L}_0^{SC} + \mathcal{L}_t. \tag{1}$$

We employ the standard formalism of Ref. [11, 12, 13] for the singlet FQH edge states and focus on the primary spin-singlet Halperin states [14] (m, m, n) (with $n = m - 1$, m odd and filling factor $\nu_\pm = 2/(2n \pm 1)$) not only for the sake of simplicity but also since these states have been observed experimentally [1, 2, 3], although these ideas can be extended to the Jain-like hierarchy of singlet FQH states [13, 15]. These edge states can be described solely in terms of one propagating chiral boson per each spin component or, equivalently, in terms of a charge chiral boson ϕ_c and a spin chiral boson ϕ_s. For the Halperin primary states with filling factor $\nu_\pm = 2/(2n \pm 1)$, the resulting edge effective Lagrangian is

$$\begin{aligned} \mathcal{L}_0^{FQH} =& \frac{1}{4\pi} \partial_1 \phi^c (\partial_0 \phi^c - v_c \partial_1 \phi^c) \\ &+ \frac{1}{4\pi} \partial_1 \phi^s (\pm \partial_0 \phi^s - v_s \partial_1 \phi^s). \end{aligned} \tag{2}$$

The electron operator, constructed from quasiparticles to

have correct quantum numbers and statistics, takes the form

$$\psi^\dagger(x)_{e,\uparrow\downarrow} = \eta_{\uparrow,\downarrow} : \exp\left(\frac{i}{\sqrt{\nu}}\phi^c(x)\right) : \times : \exp\left(\pm\frac{i}{\sqrt{2}}\phi^s(x)\right) : \tag{3}$$

up to irrelevant operators [13], where $\eta_{\uparrow,\downarrow}$ are Klein factors that ensure fermionic statistics. From this we can use operator product expansions to obtain the creation operator for a charge $2e$ spin-singlet Cooper pair $\psi_{2e}^\dagger(x)$ having tunneled into the FQH side:

$$\psi_{2e}^\dagger(x) = \frac{1}{\sqrt{2\pi}} : \exp\left(i\frac{2}{\sqrt{\nu}}\phi^c(x)\right) : \tag{4}$$

We now turn to the SC side and consider the aforementioned possibilities of the superconducting phase being either (i) pinned or (ii) dynamic. Whether or not the quantum dynamics of the superconductor needs to be taken into account depends on the size of the lead, its coupling to the environment, and the nature of the contact. In principle, the phase degree of freedom in the SC ought to exhibit quantum mechanical fluctuations and should be treated dynamically. However, equilibration with the external leads causes dissipation leading to the pinning of the phase of the superconductor [16].

When we take the phase fluctuation into consideration, we describe the Cooper pair wave function in the superconducting side in terms of phase excitations with charge $2e$. Reasoning along the lines of Ref. [17], one finds that only one channel for Cooper pairs couples to the PC and that the Cooper pair operators can be described in terms of a single chiral boson $\Phi(x)$ on an infinite line with Lagrangian density

$$\mathcal{L}_0^{SC} = \frac{1}{4\pi} \partial_1 \Phi (\partial_0 \Phi - v_{sc} \partial_1 \Phi) \tag{5}$$

with the Cooper pair operator given in terms of $\Phi(x)$ by

$$\Psi_{2e}(x) = \frac{1}{\sqrt{2\pi}} : e^{i\sqrt{2}\Phi(x)} : \tag{6}$$

to have charge $2e$ and bosonic statistics. On the other hand, when the coupling to the environment is significant enough to pin the phase of the superconductor, the superconducting order parameter acquires a finite expectation value, and the Cooper pair operator Ψ_{2e} on the superconducting side can be replaced by its expectation value $\langle \Psi_{2e} \rangle$. Then the SC effectively acts as a reservoir of Cooper pairs that can tunnel into the FQH edge states with the tunnel coupling proportional to $\langle \Psi_{2e} \rangle$.

Turning now to the tunneling contribution to the Lagrangian density, all the processes shown in Fig.1 can be described by the tunneling density

$$\mathcal{L}_t = \Gamma_{ab} \delta(x) e^{ie(V_a + V_b)t/\hbar} (\psi_{a\uparrow}^\dagger \psi_{b\downarrow}^\dagger - (\uparrow\leftrightarrow\downarrow)) \Psi_{2e} + h.c., \tag{7}$$

where $\psi_{aj}^\dagger$ is the electron creation operator in edge state a and with spin j. The incoming edge-state a is assumed

to be held at a potential drop V_a with respect to the superconductor and similarly for the edge state b. The tunneling events in Fig.1 are of two types- tunneling Γ wherein whole pairs tunnel into one edge state, and $\tilde{\Gamma}$ where Cooper pairs split into two edge states, $i.$ $e.$

$$\Gamma \equiv \Gamma_{aa}; \quad \tilde{\Gamma} \equiv \Gamma_{ab}, \quad a \neq b, \qquad (8)$$

where for convenience, we have assumed that all tunneling strengths for each kind of process are equal, but our results can be generalized straightforwardly when this is not the case. When $a = b$, the Cooper pair tunneling amounts to the creation of a whole pair in one edge represented by the operator $\psi_{2e}^{\dagger}$ of Eq.(4) with $e(V_a + V_b) = 2eV$, reflecting the fact that tunneling involves charge $2e$ Cooper pair processes. On the other hand, when $a \neq b$, Eq.(7) describes the splitting of Cooper pairs into two electrons tunneling into separate edge states, but retaining the spin-singlet phase correlations of the Cooper pair.

In the case of Set-up I (Fig.1a), the Cooper pair tunnelling amplitude (into their only available edge state) is given by Γ. However, even for Set-up II (Fig.1b,c), the only significant tunneling process in the UV limit is from the SC to the closest (say, right-moving) edge-state, once again described by the amplitude Γ. However, the presence of the SC lead necessarily depletes the right-moving edge towards the left-moving edge, providing a matrix element for inter-edge quasiparticle tunneling processes at the PC. Since such tunneling processes in a Hall bar are relevant perturbations, there exists a crossover energy scale Δ_{split} below which quasiparticle tunneling becomes dominant [18] resulting in the splitting of the QH liquid into two separate fluids (or droplets) '1' and '2' at asymptotically low temperatures and voltages, shown in Fig.1c. The particular advantage of this setup lies in the fact that what plays the role of a back gate is a SC, into and out of which Cooper pairs can tunnel. Now, very much as in the nanotube set-up suggested in Ref. [5], tunneling currents into systems '1' and '2' each have two contributions: a) from Cooper pairs tunneling from the SC into either one of the edges of the disconnected droplets (with amplitude Γ), and b) from pairs emerging from the SC, splitting into entangled electrons which are each carried away by an edge state (with amplitude $\tilde{\Gamma}$).

Equipped with the bosonized Lagrangian given above, we can now perturbatively calculate the tunneling conductance for the various situations. In the case where the SC phase is pinned, the contribution $\mathcal{L}_0^{SC}$ drops out of the Lagrangian density of Eq.(1) and the Cooper pair operator on the SC side gets replaced by its expectation value in $\mathcal{L}_t$. When the SC dynamics is important, it is convenient to perform a rotation of bosonic fields [17]. For instance, for the Cooper pair tunneling into a single

edge-state, the new Lagrangian takes the form [17]

$$\mathcal{L} = \frac{1}{4\pi} \partial_1 \varphi (\partial_0 \varphi - \partial_1 \varphi) + \frac{1}{4\pi} \partial_1 \tilde{\varphi} (\partial_0 \tilde{\varphi} - \partial_1 \tilde{\varphi})$$
$$+ \Gamma \delta(x) \cos \left(\sqrt{1 + \frac{2}{\nu}} \; (\varphi(0,t) - \tilde{\varphi}(0,t)) \right), \qquad (9)$$

where the fields φ and $\tilde{\varphi}$ are related to the FQH edge state charge degrees of freedom ϕ_c of Eq.(2) and the SC degrees of freedom Φ of Eq.(5) by an orthogonal transformation with angle $\tan \theta = (\sqrt{2} - \sqrt{\nu})/(\sqrt{2} + \sqrt{\nu})$.

The tunneling conductances can be calculated to lowest order in tunnel amplitude [10, 17]. They show scaling behavior in T and V with an exponent γ,

$$G_t(V=0, T) \propto \left(\frac{T}{\Delta} \right)^{2\gamma}, \quad G_t(V, T=0) \propto \left(\frac{V}{\Delta} \right)^{2\gamma} \quad (10)$$

where Δ is a crossover energy scale, which depends on the process, below which the perturbative result Eq.(10) applies. For Cooper-pair tunneling we set $\Delta = T_K$ and $\gamma = \alpha$, while for split Cooper-pair tunneling $\Delta = \tilde{T}_K$, $\gamma = \beta$, where

$$T_K \sim \Gamma^{-1/\alpha}, \quad \tilde{T}_K \sim \tilde{\Gamma}^{-1/\beta}, \qquad (11)$$

The tunneling amplitudes Γ and $\tilde{\Gamma}$ are defined in Eq.(8). Here we assume a high-energy cutoff Λ set by the Landau level spacing. (For the case in which the SC phase mode is pinned, the tunneling amplitudes get renormalized by a factor of $\langle \Psi_{2e} \rangle$.) The values of the exponents α and β depend on whether or not the phase field Φ of the superconductor is dynamical or pinned by dissipation. We find $\alpha = 1/\nu - 1$, $\beta = \frac{1}{2}(1/\nu - 1)$ for the pinned case and $\alpha = 2/\nu$, $\beta = 1/\nu + 1/2$ for the dynamical case.

We now discuss the tunneling conductances associated with the specific cases shown in Fig. 1 by applying the general form of Eq.(10). The conductance arising from Cooper pair tunneling between the SC lead and a single FQH edge (Fig.1a and Fig.1b) has the form

$$G_t(V, 0) \propto \frac{2e^2}{h} \left(\frac{V}{T_K} \right)^{2\alpha}, \quad G_t(0, T) \propto \frac{2e^2}{h} \left(\frac{T}{T_K} \right)^{2\alpha} \quad (12)$$

Thus, for all values of ν and at low energy scales, a SQH-SC junction, with a superconductor whose phase is pinned, has a larger conductance compared to one with dynamics. This is expected since the states on both sides of the junction will have a smaller overlap if the phase of the SC fluctuates than if it does not. So, for instance, for the $\nu = 2/3$ SQH state, $2\alpha = 1$ for the pinned case and $2\alpha = 6$ with dynamics, whereas for the $\nu = 2/5$ SQH state, the exponents are $2\alpha = 3$ and $2\alpha = 10$ respectively. In contrast, the tunneling conductance into polarized FQH states is exponentially suppressed as one of the constituent electrons of the Cooper pair has to flip its spin in order to tunnel into the FQH state. Thus, the observation of a power law conductance is a direct signature of a singlet FQH state.

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The situation in Fig.1c, wherein quasiparticle tunneling leads to splitting up of the Hall droplet into two disconnected fluids, presents the most intriguing possibility. The tunneling conductances G_t^1 and G_t^2 associated with Cooper pairs tunneling into edge-states '1' and '2' is

$$G_t^a(V=0, T) \sim A_T \left(\frac{T}{T_K}\right)^{2\alpha} + B_T \left(\frac{T}{T_K'}\right)^{2\beta}$$
$$G_t^a(V, T=0) \sim A_V \left(\frac{2V_a}{T_K}\right)^{2\alpha} + B_V \left(\frac{V_1+V_2}{T_K'}\right)^{2\beta} \quad (13)$$

where A_T, A_V, B_T and B_V are constants.

In Eq.(13) we see that the conductance has two contributions: a) one due to pair tunneling as a whole into one edge-state, with strength $\Gamma = \Gamma_{aa}, a = 1, 2$, and associated with chemical potential $2eV_a$, and b) another due to processes, with strength $\tilde{\Gamma} = \Gamma_{12}$ associated with chemical potential $e(V_1 + V_2)$ in Eq.(7), in which the Cooper pair *splits* into its constituent electrons which tunnel separately into the two edge-states as entangled electrons. The first contribution has the same form as the conductances described in Set-up I (Fig.1a) and Set-up II (Fig.1b), while the second contribution corresponds to Set up II, Fig.1c. By comparing the exponents associated with the two contributions in Eq. (13), we find that regardless of whether the SC phase is pinned ($\alpha = \frac{1}{\nu}-1$ and $\beta = \frac{\alpha}{2}$) or dynamic ($\alpha = \frac{2}{\nu}$ and $\beta = \frac{\alpha}{2} + \frac{1}{2}$) in all cases $\alpha > \beta$, and hence at low energies the entangled process *always* dominates. For $1 < \nu < 2$ the allowed SQH states are hole-like and their edge states are prone to an edge reconstruction with a $\nu = 2$ layer being formed. In this range of filling factors the QEFP is not accessible.

What are the observable signatures of the QEFP? A direct indication for charge splitting would be the observation of an indirectly driven tunneling current, *e.g.* a non-zero current I_t^1 when $V_1 = 0$ but $V_2 \neq 0$. A more direct signature of singlet entanglement requires the measurement of spin current correlations across the two FQH edges. At the QEFP there are delicate correlations in the spin currents of the two split singlet FQH fluids which exhibit correlations similar to those in discussed in Ref. [5] (for a different physical system). If $\vec{n}_1$ and $\vec{n}_2$, with relative angle θ, denote two axes of spin polarization, the correlation functions of the spin currents $I_{i,\pm n_i}^s$, with polarization state $|\pm\vec{n}_i\rangle$ $(i = 1, 2)$ at the QEFP obey [5]

$$\langle I_{1,\pm\vec{n}_1}^s I_{2,\pm\vec{n}_2}^s \rangle = e \sin^2(\theta/2) \langle I_{2,\vec{n}_2}^s \rangle$$
$$\langle I_{1,\pm\vec{n}_1}^s I_{2,\mp\vec{n}_2}^s \rangle = e \cos^2(\theta/2) \langle I_{2,\vec{n}_2}^s \rangle \quad (14)$$

In summary, we discussed the physics of a tunnel junction of a singlet SC lead and a FQH state, and analyzed its transport properties. We discussed the power-law behaviors of the tunneling conductances in temperature and voltage which should be observable *only* for singlet FQH states, thus filtering out specific filling fractions and probing their spin polarization. We found that the low energy regime of this system is governed by a QEFP in which the Hall bar is effectively split into two QH droplets into which Cooper pairs can split and tunnel coherently as *entangled electrons*. The SQH/SC junction discussed here has a natural mechanism leading to a quantum entangled fixed point. The geometry of the pair splitting state presents analogies with those of Ref. [19] for observing quasi- particle and electron tunneling. Finally, the set-up and measurements proposed here can be used to investigate spin transitions in QH states, as well as open issues on the Luttinger liquid nature of edge states.

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